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EXPERIMENTS WITH MESH MOVING AND LOCAL REFINEMENT ALGORITHMS FOR HYPERBOLIC SYSTEMS[1]

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ABSTRACT

Computational experiments using adaptive procedures that combine mesh motion and local mesh refinement are presented for one- and two-dimensional time-dependent partial differential systems. The adaptive algorithms were developed by Arney and Flaherty [1] and involve motion of a coarse base mesh that isolates spatially distinct phenomena and local mesh refinement that recursively divides the time step and spatial cells of the moving base mesh in regions where a refinement indicator exceeds a prescribed tolerance. Numerical solutions of the Euler equations using a MacCormack finite volume scheme are presented for one- and two-dimensional shock problems.

1. Introduction.

Arney and Flaherty [1] developed an adaptive procedure for solving vector systems of time-dependent partial differential equations that combined motion of a coarse base mesh, local refinement of the base mesh in regions where a refinement indicator exceeded a prescribed tolerance, and static mesh regeneration to create a new base mesh whenever an existing one became too distorted. Their procedures were applicable in one and two dimensions and could be used in conjunction with most numerical methods. Their procedures also required specification of local motion and refinement indicators on each cell of the mesh. These indicators could, for example, be estimates of the local discretization error, solution gradients, or any function that is large where additional resolution is needed and small where less resolution is desired. They performed mesh motion by locating computational cells having large motion indicators and clustering them into isolated regions that were presumed to contain similar solution characteristics. The center of motion indicators of each clustered region was moved so as to follow the dynamics of the solution. Remaining portions of the mesh were moved according to an algebraic function so as to

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produce a smooth grid having minimal distortion. Clustered regions created at one time step could subsequently be destroyed when the dynamic phenomena subsided. Similarly, two or more clusters could be united when structures of the solution intersected.

Their results [1,2] indicated that proper mesh motion could dramatically reduce errors in some instances for a modest increase in computational cost. However, mesh motion alone cannot in general produce solutions that satisfy prescribed accuracy requirements. For this reason, they combined their mesh moving procedures with a local temporal and spatial mesh refinement strategy. In particular, they recursively divided space-time cells of a mesh by integer factors and solved sequences of local problems on successively smaller-and-smaller domains. A dynamic tree structure, where fine grids were regarded as offspring of coarser ones, was used to manage the solution, mesh, and indication data associated with the motion and refinement strategies.

Some problems, e.g., those having large material deformations, can result in tangling and severe distortion of the moving base mesh. When this occurred, Arney and Flaherty [1] invoked a static mesh regeneration procedure that created a new base mesh.

These mesh motion, static regeneration, and local refinement procedures are explicit and independent of each other as well as the numerical solution technique and motion and refinement indication data. The purpose of this work is to explore questions of optimal combinations of the various adaptive strategies for hyperbolic systems. Examples involving solutions of the Euler equations for a one-dimensional shock tube and a two-dimensional piston problem are used for this purpose. Following Arney and Flaherty [1], we use MacCormack's scheme with Davis's artificial viscosity model to discretize the Euler equations in the conservative form

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x + \mathbf{g}(\mathbf{u})_y = 0, \tag{1a}$$

where

$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad \mathbf{f}(\mathbf{u}) = \begin{bmatrix} \rho u \\ \rho u^2+p \\ \rho uv \\ u(e+p) \end{bmatrix}, \quad \mathbf{g}(\mathbf{u}) = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2+p \\ v(e+p) \end{bmatrix}. \tag{1b,c,d}$$

Here, ρ is the fluid density; u and v are the Cartesian components of the velocity vector; e is the internal energy; and subscripts denote partial differentiation with respect to time t or the spatial coordinates x and y. The pressure p is evaluated according to the ideal gas equation of state

$$p = (\gamma - 1)\rho[e - (u^2 + v^2)/2], \tag{2}$$

where γ is the specific heat ratio of the fluid. One-dimensional problems are obtained by setting $v = 0$ and $\partial(\ )/\partial y = 0$ in (1,2). The examples of Section 2 were obtained with $\gamma = 1.4$. An estimate of the local discretization error obtained by Richardson's extrapolation was used as a refinement indicator. The mesh motion indicator was taken as a density gradient on neighboring cells normalized with respect to the average densities on neighboring cells [4].

2. Computational Experiments.

Example 1. Consider Sod's [3] one-dimensional shock tube problem which consists of the Euler equations (1,2) subject to the initial conditions

$$\begin{bmatrix} \rho(x,0) \\ p(x,0) \\ u(x,0) \end{bmatrix} = \begin{cases} [1.0,\ 1.0,\ 0.0]^T, & \text{if } 0 \le x \le 0.5 \\ [0.125,\ 0.1,\ 0.0]^T, & \text{if } 0.5 < x \le 1.5 \end{cases} \tag{3}$$

A diaphragm at $x = 0.5$ separates two regions of a tube that contain gases at different densities and pressures. At time $t = 0$ the diaphragm is ruptured and three waves are generated: a shock moving with velocity 1.7522, a contact discontinuity moving with speed 0.9275, and an expansion wave centered between $0.5 - 1.1832t \le x \le 0.5 - 0.0703t$.

We solved this problem on $0 \le x \le 1.5$ for $0 \le t \le 0.35$ using base meshes of 16 elements with (i) mesh motion, (ii) local refinement, and (iii) mesh motion and local refinement. Refinement was restricted to a maximum of four levels to avoid excessive refinement near shocks. The computational effort of the local refinement scheme is presented as a function of the local discretization error tolerance in Table 1. The CPU time and number of space-time cells used to solve the problem are recorded as measures of complexity. Global errors in the L_1 at $t = 0.35$ are also tabulated. For small tolerances, CPU times and the number of space-time cells increase approximately linearly with decreasing L_1 error. The decrease in the local pointwise error tolerance is slightly sublinear when compared with the actual global L_1 error.

Error Tolerance	CPU Time	Space-Time Cells	$\|e\|_1 \times 10^3$
0.00512	284	368	22.4
0.00128	763	1632	13.0
0.00032	2235	6452	5.51
0.00008	4694	15568	1.89

Table 1. CPU time, number of space-time cells, and L_1 error at $t = 0.35$ as a function of the local error tolerance for Example 1.

Strategy	CPU Time	Space-Time Cells	$\|e\|_1 \times 10^3$
Mesh Motion	587	704	19.4
Refinement	4694	15568	1.89
Motion & Refin.	10269	30608	1.51

Table 2. CPU time, number of space-time cells, and L_1 error at $t = 0.35$ for solutions of Example 1 obtained with (i) mesh motion, (ii) local refinement, and (iii) mesh motion and local refinement.

Similar data is presented in Table 2 for the three adaptive strategies. The meshes used for these computations are presented in Figure 1. A local error tolerance of 0.00008

was used in conjunction with the refinement indicators. The moving mesh procedure follows the features of the solution, but the mesh is too coarse to obtain good resolution. Refinement was correctly performed at all critical points of the calculation. In each case, shocks are captured sharply with the correct speed. As expected, diffusive effects are more pronounced near the contact surface. The results suggest that mesh motion, with or without refinement, is not competitive with refinement alone. Additional experimentation is needed to determine a better combination of mesh moving and refinement.

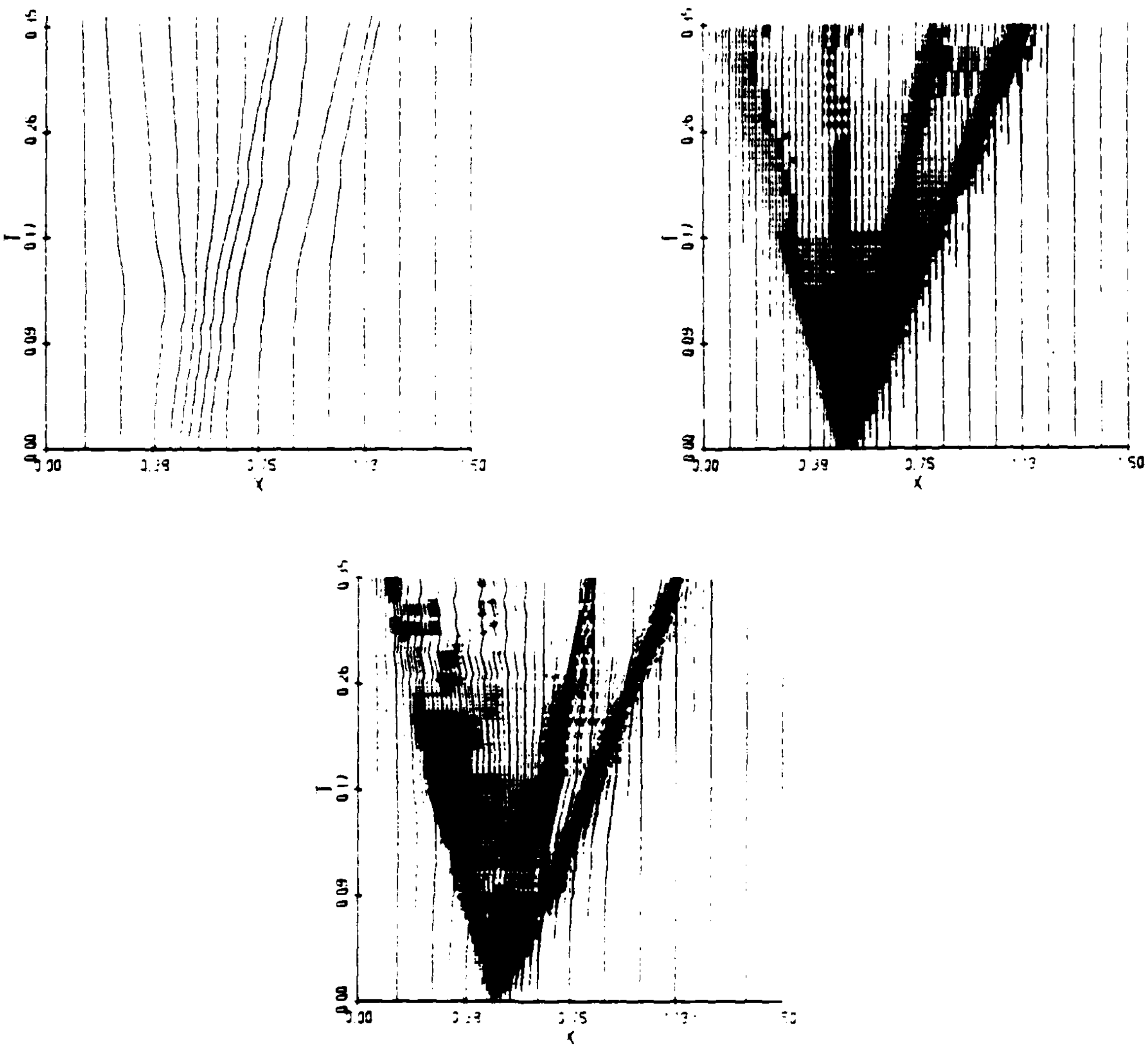

Figure 1. Meshes for computations performed with mesh motion (upper left), local refinement (upper right), and mesh motion and local refinement (bottom) for Example 1.

Example 2. Consider the solution of the Euler equations (1,2) in a region exterior to an infinite cylindrical piston that is expanding radially creating a radially expanding shock wave. We ignore the cylindrical symmetry and solve this problem in one quadrant of the rectangular domain $-0.05 \leq x,y \leq 0.05$. Self similar exact solutions of this problem may

be obtained by solving a pair of ordinary differential equations (by numerical integration) for the radial velocity and acoustic speed [4].

We solved this problem for $0 < t \leq 0.0096$ with the piston initially at a radius of 0.016023 and having a velocity of 1.6185. Numerical solutions were calculated on a 26×26 mesh (i) without adaptation, (ii) with one level of local refinement, and (iii) with mesh motion and one level of refinement. Contours of the density at $t = 0.0096$ are presented for the exact and three numerical solutions in Figure 2. The meshes produced by the two adaptive strategies at $t = 0.0096$ are shown in Figure 3.

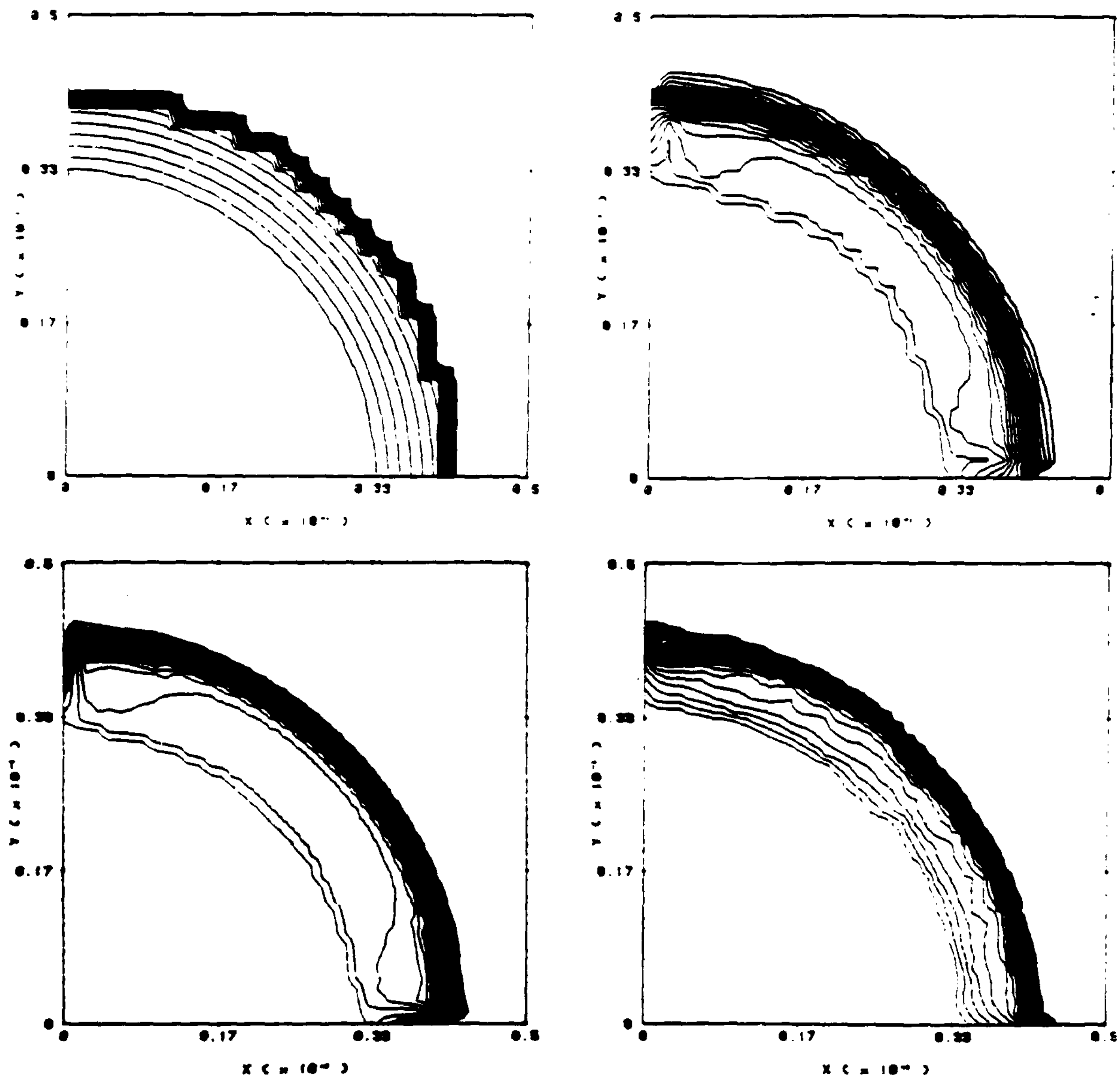

Figure 2. Density contours for Example 2 at $t = 0.0096$ obtained from the exact solution (upper left) and by computed solutions on a uniform stationary mesh (upper right), a uniform stationary base mesh with one level of refinement (lower left), and a moving base mesh with one level of refinement (lower right).

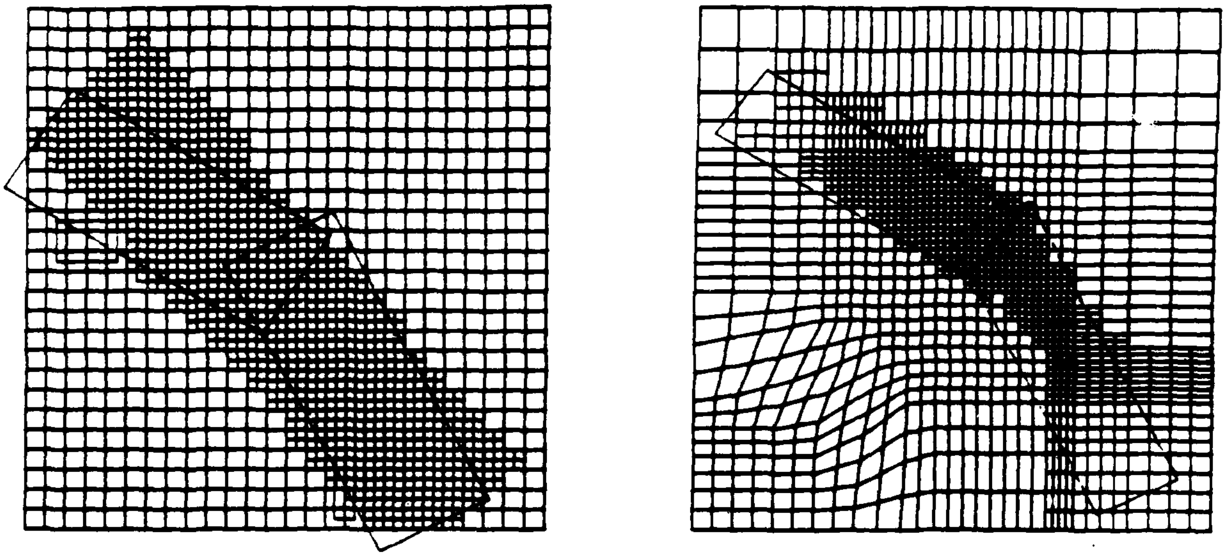

Figure 3. Spatial meshes at $t = 0.0096$ for Example 2 using one level of local mesh refinement on a uniform stationary base mesh (left) and a moving base mesh (right).

Clearly one level of refinement is not sufficient to adequately resolve the structure of this solution. We were forced to limit our computations to this level because of memory restrictions on our computing system. Nevertheless, local refinement with and without mesh moving provide improvements over uniform stationary mesh solutions. Detailed quantitative comparisons are yet to be performed; however, qualitatively, the expanding shock is sharper in both adaptive solutions. The combination of mesh motion and refinement provides additional improvement.

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